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Three family type IIB orientifold string vacua with non-abelian Wilson lines

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Abstract: We address the implementation of non-abelian Wilson lines in $D = 4$ $N = 1$ type IIB orientifold constructions. We present an explicit three-family example with the gauge group $(\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2))^2 \times (\mathrm{U}(6) \times \mathrm{Sp}(4))^2$ and give the particle spectrum and the trilinear superpotential. Emphasizing the new subtleties associated with the introduction of non-abelian Wilson lines, we show that the abelian gauge anomalies are cancelled by the Green-Schwarz-type mechanism, and calculate the Fayet-Iliopoulos terms and gauge coupling corrections. The analysis thus sets a stage for further investigations of the phenomenological implications of this model.

Keywords: Superstring Vacua, Anomalies in Field and String Theories.

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Contents

1. Introduction

Four-dimensional $N = 1$ supersymmetric type-IIB orientifolds [1, 2, 3, 4] provide a domain of perturbative string vacua whose study of physics implications is still at the early stages of investigation [5, 6]. In particular, there is the need to further explore the existence of string vacua with quasi-realistic features, i.e. those with gauge group close to that of the standard model and massless spectra with three families.[1] In order to reduce the gauge group structure, one mechanism involves the introduction of Wilson lines [4, 11]. (Other mechanisms involve for example the blowing-up procedure [12] or the introduction of non-zero NS-NS two-form background fields [13].)

[1] Recent explorations involve the construction of type-IIB orientifolds with branes *and* anti-branes which break supersymmetry, while keeping the models stable (tachyon-free [7, 8, 9]). A number of quasi-realistic models were recently constructed in [10].

In particular, the introduction of Wilson lines that do not commute with the discrete orbifold symmetry seems to be a promising mechanism to reduce the gauge group structure and lead us a step closer to the construction of quasi-realistic models [11]. We address the consistent implementation of the symmetry actions of both the discrete orientifold group and the Wilson line actions. In addition, constraints arising from tadpole cancellation as well as the requirement that the massless sector is free of non-abelian anomalies, severely restrict the allowed solutions.

We focus on a particular three family $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ type-IIB orientifold model [11] (a descendant of the $\mathbb{Z}_3$-orientifold [1], the first $N = 1$ supersymmetric type-IIB orientifold with three families and the gauge group $\mathrm{U}(12) \times \mathrm{SO}(8)$). We address the possible Wilson lines, which necessarily commute with the $\mathbb{Z}_3$ but not all the $\mathbb{Z}_2$ orbifold actions. We find that the consistency conditions (the absence of tadpoles and non-abelian gauge anomalies) allow for a very limited number of possible models. We focus on the (only) example with gauge group structure that encompasses that of the standard model and present the massless spectrum and the trilinear superpotential[2] (see section 2). In section 3, we explicitly demonstrate that the abelian gauge anomalies of the massless spectrum are cancelled by a Green-Schwarz-type mechanism (proposed in [16] and confirmed in [18]); in particular, we explore the additional subtleties that are associated with non-abelian Wilson lines. In section 4 we derive the form of the Fayet-Iliopoulos (FI) terms as well as the corrections to the gauge kinetic functions due to the blowing-up moduli. In section 4 we summarize results and point out possible generalizations to address a general set-up for a consistent construction of type-IIB orientifold solutions with non-abelian Wilson lines. In appendix A we derive the massless matter spectrum and in appendix B the moduli of $\mathbb{Z}_2$ twisted sectors.

2. Orientifold constructions

2.1 $D = 4$, $N = 1$ orientifolds

We compactify type-IIB theories on a six-torus T^6 and mod out a discrete symmetry group G_1 and the world-sheet parity operation Ω, which could be accompanied by another discrete symmetry G_2, i.e. the orientifold group is $G = G_1 + \Omega G_2$. Closure requires $\Omega g\, \Omega g' \in G_1$ for $g, g' \in G_2$. In the following we always have $G_1 = G_2$.

The compactified tori are described by complex coordinates X_i, $i = 1, 2, 3$. The action of an orbifold group $\mathbb{Z}_N$ on the compactified dimensions can be summarized in a twist vector $v = (v_1, v_2, v_3)$,

$$g : X_i \to e^{2i\pi v_i} X_i\,, \tag{2.1}$$

where the v_i's are multiples of $1/N$. In our conventions, $\mathcal{N} = 1$ supersymmetry requires that $v_1 + v_2 + v_3 = 1$.

[2] The presented study builds on earlier work [11]. However, we find a different gauge group and massless spectrum.

Since the Ω projection relates left- and right-movers, it gives rise to open strings [14]. Tadpole cancellation then requires the inclusion of an even number of D9-branes and additional discrete symmetries in the orientifold group may require the presence of multiple sets of D5-branes for consistency.

An open string state can be written as $|\Psi, ij\rangle$ where Ψ denotes the world-sheet state and i, j the Chan-Paton states of the left and right end points on a D9- or D5-brane. The elements $g \in G_1$ act on open string states as follows:

$$g : |\Psi, ij\rangle \to (\gamma_g)_{ii'} \, |g \cdot \Psi, i'j'\rangle \left(\gamma_g^{-1}\right)_{j'j} \, . \tag{2.2}$$

Similarly, the elements of ΩG_1 act as

$$\Omega g : |\Psi, ij\rangle \to (\gamma_{\Omega g})_{ii'} \, |\Omega g \cdot \Psi, j'i'\rangle \left(\gamma_{\Omega g}^{-1}\right)_{j'j} \, , \tag{2.3}$$

where we have defined $\gamma_{\Omega g} = \gamma_g \gamma_\Omega$, up to a phase, in accordance with the usual rules for multiplication of group elements. Note that Ω exchanges the Chan-Paton indices. Hence, $(\Omega g)^2$ acts as

$$(\Omega g)^2 : |\Psi, ij\rangle \to \left[\gamma_{\Omega g} \left(\gamma_{\Omega g}^{-1}\right)^{\mathrm{T}}\right]_{ii'} \left|(\Omega g)^2 \cdot \Psi, i'j'\right\rangle \left[\gamma_{\Omega g}^{\mathrm{T}} \gamma_{\Omega g}^{-1}\right]_{j'j} \, . \tag{2.4}$$

Since the γ_g form a projective representation of the orientifold group, consistency with group multiplication implies some conditions on the γ_g. Consider the $G_1 = G_2 = \mathbb{Z}_N$ case. If g is the generator of $\mathbb{Z}_N$, we must have

$$\gamma_g^N = \pm 1 \, . \tag{2.5}$$

Analogously, $\Omega^2 = 1$ implies that

$$\gamma_\Omega = \pm \gamma_\Omega^{\mathrm{T}} \, . \tag{2.6}$$

Further, due to the closure relation $\Omega g \, \Omega g' \in G_1$ for $g, g' \in G_2$, one must have

$$\left(\gamma_g^k\right)^* = \pm \gamma_\Omega^* \, \gamma_g^k \, \gamma_\Omega \, . \tag{2.7}$$

It turns out that the tadpoles cancel if we choose the plus sign for D9-branes and the minus sign for D5-branes [4].

In the D9-brane sector γ_Ω is symmetric and can be chosen real, while it is anti-symmetric in the D5-brane sectors and can be chosen imaginary there. Hence, in a suitable basis

$$\gamma_{\Omega,9} = \begin{pmatrix} 0 & \mathbb{1}_{16} \\ \mathbb{1}_{16} & 0 \end{pmatrix} \qquad \text{and} \qquad \gamma_{\Omega,5} = \begin{pmatrix} 0 & -i\mathbb{1}_{16} \\ i\mathbb{1}_{16} & 0 \end{pmatrix} , \tag{2.8}$$

where the subscript 9 or 5 denotes the brane sector in which these matrices are acting. Hence, in both the D9- and D5-brane sectors eq. (2.7) yields

$$\left(\gamma_g^k\right)^* = \gamma_\Omega \, \gamma_g^k \, \gamma_\Omega \, . \tag{2.9}$$

Further, finiteness of string loop diagrams yields tadpole cancellation conditions which constrain the traces of γ_g matrices. For example, in the case of $\mathbb{Z}_3$ [1]

$$\mathrm{Tr}(\gamma_{\mathbb{Z}_3}) = -4\,. \tag{2.10}$$

Open string states, whose Chan-Paton matrices will be denoted by $\lambda^{(i)}$, $i = 0, \ldots, 3$ in the following, give rise to space-time gauge bosons ($i = 0$) and matter states ($i = 1, 2, 3$). Gauge bosons in the D9-brane sector arise from open strings beginning and ending on D9-branes (the matter states will be discussed in appendix A). Invariance of these states under the action of the orientifold group requires

$$\lambda^{(0)} = -\gamma_{\Omega,9}\,{\lambda^{(0)}}^{\mathrm{T}}\gamma_{\Omega,9}^{-1} \qquad \text{and} \qquad \lambda^{(0)} = \gamma_{g,9}\,\lambda^{(0)}\gamma_{g,9}^{-1}\,. \tag{2.11}$$

With eq. (2.8) the first constraint implies that the $\lambda^{(0)}$ are SO(32) generators, while the constraints from the $\gamma_{g,9}$ will further reduce the group.

Similarly, in the D5-brane sectors the Chan-Paton matrices of the gauge bosons satisfy the following constraints:

$$\lambda^{(0)} = -\gamma_{\Omega,5}\,{\lambda^{(0)}}^{\mathrm{T}}\gamma_{\Omega,5}^{-1} \qquad \text{and} \qquad \lambda^{(0)} = \gamma_{g,5_i}\,\lambda^{(0)}\gamma_{g,5_i}^{-1}\,. \tag{2.12}$$

Due to the symplectic nature of $\gamma_{\Omega,5}$ in eq. (2.8), world-sheet parity yields Sp(32) generators for the gauge group, which will be further reduced by the additional orientifold actions.

Let us now consider the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold, first constructed in ref. [11]. Tadpole cancellation implies that we have 32 D9-branes and three different sets of 32 D5-branes, which we will distinguish by an index i. The D5_i-brane fills the 4-dimensional spacetime and the torus parametrized by X_i. The $\mathbb{Z}_3$ twist action on the tori is given by the twist vector $v = (1/3, 1/3, 1/3)$ and, both in the D9-brane and the D5_i-brane sectors, its action on Chan-Paton matrices is generated by

$$\gamma_{\mathbb{Z}_3} = \mathrm{diag}\left(\omega\mathbb{1}_{12}, \mathbb{1}_4, \omega^{-1}\mathbb{1}_{12}, \mathbb{1}_4\right), \quad \text{where } \omega = e^{2\pi i/3}\,. \tag{2.13}$$

This choice satisfies eqs. (2.9) and (2.10).

The two $\mathbb{Z}_2$'s act on the compactified coordinates X_i ($i = 1, 2, 3$) in the following way:

$$R_1 : X_1 \to X_1\,, \quad X_{2,3} \to -X_{2,3} \Longrightarrow v_{R_1} = \left(0, \frac{1}{2}, \frac{1}{2}\right), \tag{2.14}$$

$$R_2 : X_2 \to X_2\,, \quad X_{1,3} \to -X_{1,3} \Longrightarrow v_{R_2} = \left(\frac{1}{2}, 0, \frac{1}{2}\right). \tag{2.15}$$

JHEP04(2000)004

9	$\gamma_{R_1,9} = \mathrm{diag}\,(i\,\sigma_1 \otimes \mathbb{1}_6, i\,\sigma_1 \otimes \mathbb{1}_2, -i\,\sigma_1 \otimes \mathbb{1}_6, -i\,\sigma_1 \otimes \mathbb{1}_2)$ $\gamma_{R_2,9} = \mathrm{diag}\,(i\,\sigma_2 \otimes \mathbb{1}_6, i\,\sigma_2 \otimes \mathbb{1}_2, i\,\sigma_2 \otimes \mathbb{1}_6, i\,\sigma_2 \otimes \mathbb{1}_2)$ $\gamma_{R_3,9} = \mathrm{diag}\,(i\,\sigma_3 \otimes \mathbb{1}_6, i\,\sigma_3 \otimes \mathbb{1}_2, -i\,\sigma_3 \otimes \mathbb{1}_6, -i\,\sigma_3 \otimes \mathbb{1}_2)$
5_1	$\gamma_{R_1,5_1} = \mathrm{diag}\,(i\sigma_1 \otimes \mathbb{1}_6, i\,\sigma_1 \otimes \mathbb{1}_2, -i\,\sigma_1 \otimes \mathbb{1}_6, -i\,\sigma_1 \otimes \mathbb{1}_2)$ $\gamma_{R_2,5_1} = \mathrm{diag}\,(\sigma_2 \otimes \mathbb{1}_6, \sigma_2 \otimes \mathbb{1}_2, -\sigma_2 \otimes \mathbb{1}_6, -\sigma_2 \otimes \mathbb{1}_2)$ $\gamma_{R_3,5_1} = \mathrm{diag}\,(\sigma_3 \otimes \mathbb{1}_6, \sigma_3 \otimes \mathbb{1}_2, \sigma_3 \otimes \mathbb{1}_6, \sigma_3 \otimes \mathbb{1}_2)$
5_2	$\gamma_{R_1,5_2} = \mathrm{diag}\,(\sigma_1 \otimes \mathbb{1}_6, \sigma_1 \otimes \mathbb{1}_2, \sigma_1 \otimes \mathbb{1}_6, \sigma_1 \otimes \mathbb{1}_2)$ $\gamma_{R_2,5_2} = \mathrm{diag}\,(i\,\sigma_2 \otimes \mathbb{1}_6, i\,\sigma_2 \otimes \mathbb{1}_2, i\,\sigma_2 \otimes \mathbb{1}_6, i\,\sigma_2 \otimes \mathbb{1}_2)$ $\gamma_{R_3,5_2} = \mathrm{diag}\,(\sigma_3 \otimes \mathbb{1}_6, \sigma_3 \otimes \mathbb{1}_2, \sigma_3 \otimes \mathbb{1}_6, \sigma_3 \otimes \mathbb{1}_2)$
5_3	$\gamma_{R_1,5_3} = \mathrm{diag}\,(\sigma_1 \otimes \mathbb{1}_6, \sigma_1 \otimes \mathbb{1}_2, \sigma_1 \otimes \mathbb{1}_6, \sigma_1 \otimes \mathbb{1}_2)$ $\gamma_{R_2,5_3} = \mathrm{diag}\,(-\sigma_2 \otimes \mathbb{1}_6, -\sigma_2 \otimes \mathbb{1}_2, \sigma_2 \otimes \mathbb{1}_6, \sigma_2 \otimes \mathbb{1}_2)$ $\gamma_{R_3,5_3} = \mathrm{diag}\,(i\,\sigma_3 \otimes \mathbb{1}_6, i\,\sigma_3 \otimes \mathbb{1}_2, -i\,\sigma_3 \otimes \mathbb{1}_6, -i\,\sigma_3 \otimes \mathbb{1}_2)$

Table 1: Representation of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ twist matrices in the D9- and D5$_i$-brane sectors.

The corresponding γ-matrices have to fulfill certain group consistency conditions [2]:

$$\gamma^{\mathrm{T}}_{R_i\Omega,s} = -\mathcal{C}_{i,s}\gamma_{R_i\Omega,s}\,, \tag{2.16}$$

$$\gamma_{R_i\Omega,s}\gamma^{-1\,\mathrm{T}}_{R_j\Omega,s}\gamma_{R_k\Omega,s}\gamma^{-1\,\mathrm{T}}_{\Omega,s} = \mathcal{C}_{0,s}\mathcal{C}_{3,s}\epsilon_{ijk}\,, \tag{2.17}$$

$$\gamma_{R_i\Omega,s}\gamma^{-1\,\mathrm{T}}_{\Omega,s}\gamma_{R_i\Omega,s}\gamma^{-1\,\mathrm{T}}_{\Omega,s} = \mathcal{C}_{0,s}\mathcal{C}_{i,s}\,, \tag{2.18}$$

$$\gamma_{R_i,s}\gamma_{R_j,s}\gamma_{R_k,s} = -\mathcal{C}_{0,s}\mathcal{C}_{3,s}\epsilon_{ijk}\,, \tag{2.19}$$

$$\gamma_{R_i,s}\gamma_{R_i,s} = \mathcal{C}_{0,s}\mathcal{C}_{i,s}\,, \tag{2.20}$$

where $s = 0$ and $s = 1,2,3$ denote D9-branes and D5$_s$-branes, respectively. The co-cycle $\mathcal{C}_{i,s}$ is equal to -1 for $i = s$ and to $+1$ otherwise. Further, tadpole cancellation implies

$$\mathrm{Tr}\,(\gamma_{R_i,s}) = \mathrm{Tr}\,(\gamma_{R_i,s}\gamma_{\mathbb{Z}_3}) = 0\,. \tag{2.21}$$

One set of γ_{R_i} matrices fulfilling these constraints is given in table 1.

Consider now the Chan-Paton matrix for the gauge fields in the D9-brane sector. Inserting $\gamma_{\mathbb{Z}_3,9}$ and the $\gamma_{R_i,9}$ in eq. (2.11) yields the following form for $\lambda^{(0)}$:

$$\lambda^{(0)} = \begin{pmatrix} \mathbb{1}_2 \otimes B_1 & 0 & 0 & 0 \\ 0 & \mathbb{1}_2 \otimes B_2 & 0 & i\sigma_2 \otimes S_1 \\ 0 & 0 & -\mathbb{1}_2 \otimes B_1^{\mathrm{T}} & 0 \\ 0 & -i\sigma_2 \otimes S_2 & 0 & -\mathbb{1}_2 \otimes B_2^{\mathrm{T}} \end{pmatrix}. \tag{2.22}$$

Here, B_1 is a general 6×6 matrix, corresponding to the adjoint representation of U(6). B_2 is a general 2×2 matrix and S_1 and S_2 are symmetric 2×2 matrices, i.e. B_2, S_1 and S_2 form an adjoint representation of Sp(4).

In the D5_i-brane sectors, the Chan-Paton matrices for the gauge fields have a similar structure, i.e. we get three additional copies of the gauge group $\mathrm{U}(6) \times \mathrm{Sp}(4)$.

2.2 Wilson lines

In order to further break the gauge symmetry we introduce discrete Wilson lines, which can be written as a matrix γ_W acting on the Chan-Paton matrices. A Wilson line along the two-torus X_i has to satisfy the following algebraic consistency conditions:

$$\left(\gamma_{\mathbb{Z}_3}\gamma_W\right)^3 = +1\,, \qquad \left(\gamma_{R_j,9}\gamma_W\right)^2 = -1\,, \qquad \left(\gamma_{R_j,5_i}\gamma_W\right)^2 = +1 \quad \text{for } j \neq i\,. \tag{2.23}$$

These conditions and eqs. (2.19) and (2.20) then imply that γ_W and γ_{R_i} have to commute in the D9- and D5_i-brane sectors,

$$[\gamma_{R_i,9}, \gamma_W] = 0\,, \qquad [\gamma_{R_i,5_i}, \gamma_W] = 0\,. \tag{2.24}$$

This is due to the fact that γ_{R_i} does not act on the compactified coordinate X_i (cf. eqs. (2.14) and (2.15)). Further, tadpole cancellation requires

$$\mathrm{Tr}\left(\gamma_{\mathbb{Z}_3}\right) = \mathrm{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W\right) = \mathrm{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W^2\right) = -4\,, \tag{2.25}$$

$$\mathrm{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W\gamma_{R_j,s}\right) = \mathrm{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W^2\gamma_{R_j,s}\right) = 0 \quad \text{for } s = 9 \text{ or } 5_i\,. \tag{2.26}$$

In the following we will restrict ourselves to the case of one Wilson line along the third two-torus. It turns out that eqs. (2.23)–(2.26) allow only one Wilson line (up to equivalent representations) which yields a gauge group containing the standard model as a subgroup:

$$\gamma_W = \mathrm{diag}\left(W_1, W_2, W_1^{-1}, W_2^{-1}, W_3, W_3^{-1}, W_1^{-1}, W_2^{-1}, W_1, W_2, W_3^{-1}, W_3\right), \tag{2.27}$$

where

$$W_1 = \mathrm{diag}\left(\omega, \omega, 1\right), \qquad W_2 = \mathbb{1}_3\,, \qquad W_3 = \mathrm{diag}\left(\omega, 1\right). \tag{2.28}$$

Since we place all the D5-branes at the origin, this Wilson line acts only in the D9- and D5_3-brane sectors. The gauge bosons in these two sectors have to obey one additional constraint,

$$\lambda^{(0)} = \gamma_W \lambda^{(0)} \gamma_W^{-1}\,. \tag{2.29}$$

The generator B_1 of the U(6) in eq. (2.22) therefore gets reduced to

$$B_1 = \begin{pmatrix} \tilde{B}_1 & 0 \\ 0 & \tilde{B}_2 \end{pmatrix}, \tag{2.30}$$

where $\tilde{B}_1$ and $\tilde{B}_2$ are general 2×2 and 4×4 matrices, respectively. Hence, the U(6) gets broken down to a $\mathrm{U}(4) \times \mathrm{U}(2)$. Similarly, the Sp(4) generators B_2 and S_i are reduced to

$$B_2 = \begin{pmatrix} W_1 & 0 \\ 0 & Z_1 \end{pmatrix}, \qquad S_1 = \begin{pmatrix} W_2 & 0 \\ 0 & Z_2 \end{pmatrix}, \qquad S_2 = \begin{pmatrix} W_3 & 0 \\ 0 & Z_3 \end{pmatrix}. \tag{2.31}$$

Here, the W_i and Z_i generate two SU(2)'s.

In the 5_1 and 5_2 sectors the gauge groups remain unbroken. The action of Wilson lines on the matter states is discussed in appendix A and the resulting spectrum of the $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ model with one Wilson line is given in table 2.

The tree-level Yukawa superpotential of the massless states reads[3]

$$\begin{aligned}
\mathcal{W} = & \sum_{s=0,1,2,3} \sum_{i \neq j \neq k} \chi_i^{(s)} \psi_j^{(s)} \psi_k^{(s)} + \chi_i^{(0)} S^{(0i)} S^{(0i)} + \varepsilon_{ijk} \chi_i^{(j)} S^{(jk)} S^{(jk)} + \\
& + \psi_i^{(0)} Q^{(0i)} U^{(0i)} + \varepsilon_{ijk} \psi_i^{(j)} Q^{(jk)} U^{(jk)} + \varepsilon_{ij3} \chi_i^{(j)} U^{(3j)} U^{(3j)} + \\
& + \varepsilon_{ij3} \chi_i^{(j)} T^{(3j)} T^{(3j)} + \varepsilon_{ij3} \psi_i^{(j)} P^{(3j)} R^{(3j)} + \varepsilon_{ij3} \psi_i^{(j)} Q^{(3j)} S^{(3j)} + \\
& + \chi_i^{(i)} U^{(0i)} U^{(0i)} + \chi_i^{(i)} T^{(0i)} T^{(0i)} + \psi_i^{(i)} P^{(0i)} R^{(0i)} + \psi_i^{(i)} Q^{(0i)} S^{(0i)} + \\
& + \sum_{s=0,3} \left(P^{(si)} R^{(sj)} S^{(ij)} + Q^{(si)} S^{(sj)} S^{(ij)} + T^{(si)} T^{(sj)} Q^{(ij)} + U^{(si)} U^{(sj)} Q^{(ij)} \right),
\end{aligned} \tag{2.32}$$

where summation over repeated indices is understood and we defined $S^{(ij)} \equiv U^{(ji)}$. Further, we have suppressed the actual Yukawa couplings and determined only trilinear couplings. Note that $\eta^{(s)}$ and $\tilde{\eta}^{(s)}$, the extra matter states from $\lambda^{(3)}$ in the 9 and 5_3 sectors (cf. appendix A), do not have trilinear couplings.

3. Chern-Simons terms and anomaly cancellation

It is obvious from the spectrum that the U(1) factors in various sectors are anomalous at the effective field theory level. Therefore, it is important to check that the abelian anomalies cancel through the generalized Green-Schwarz mechanism [16].

Before we demonstrate the cancellation of the U(1) anomalies explicitly, we turn to a discussion of the NS-NS sector moduli fields of the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold. There are three untwisted sector moduli associated with the scaling deformation of each two-torus. If we denote the complexified bosonic coordinate associated with the J-th two-torus as X^J, the three moduli fields are represented by $\partial_z X^J \partial_{\bar{z}} \bar{X}^J$ $(J = 1, 2, 3)$ vertices.

[3] The non-zero terms in the superpotential are determined by gauge invariance and standard techniques of superconformal field theory in the closed superstring sector [15].

Sector	Gauge group	Field	Representation
99	$U(4) \times U(2) \times SU(2) \times SU(2)$	$\chi_k^{(0)}$	$3 \times (6,1,1,1)(+2,0)$
		$\psi_k^{(0)}$	$3 \times (\bar{4},1,1,2)(-1,0)$
		$\eta^{(0)}$	$(1,1,1,1)(0,+2)$
		$\tilde{\eta}^{(0)}$	$(1,2,2,1)(0,-1)$
$5_i 5_i$	$U(6) \times Sp(4)$	$\chi_k^{(i)}$	$3 \times (15,1)(+2)$
$i=1,2$		$\psi_k^{(i)}$	$3 \times (\bar{6},4)(-1)$
$5_3 5_3$	$U(4) \times U(2) \times SU(2) \times SU(2)$	$\chi_k^{(3)}$	$3 \times (6,1,1,1)(+2,0)$
		$\psi_k^{(3)}$	$3 \times (\bar{4},1,1,2)(-1,0)$
		$\eta^{(3)}$	$(1,1,1,1)(0,+2)$
		$\tilde{\eta}^{(3)}$	$(1,2,2,1)(0,-1)$
95_i	$[U(4) \times U(2) \times SU(2) \times SU(2)]_9$	$P^{(0\,i)}$	$(1,2,1,1;6,1)(0,+1;+1)$
$i=1,2$	$\times [U(6) \times Sp(4)]_{5_i}$	$Q^{(0\,i)}$	$(4,1,1,1;6,1)(+1,0;+1)$
		$R^{(0\,i)}$	$(1,2,1,1;1,4)(0,-1;0)$
		$S^{(0\,i)}$	$(\bar{4},1,1,1;1,4)(-1,0;0)$
		$T^{(0\,i)}$	$(1,1,2,1;\bar{6},1)(0,0;-1)$
		$U^{(0\,i)}$	$(1,1,1,2;\bar{6},1)(0,0;-1)$
95_3	$[U(4) \times U(2) \times SU(2) \times SU(2)]_9$	$Q^{(0\,3)}$	$(4,1,1,1;4,1,1,1)(+1,0;+1,0)$
	$\times [U(4) \times U(2) \times SU(2) \times SU(2)]_{5_3}$	$S^{(0\,3)}$	$(\bar{4},1,1,1;1,1,1,2)(-1,0;0,0)$
		$U^{(0\,3)}$	$(1,1,1,2;\bar{4},1,1,1)(0,0;-1,0)$
$5_1 5_2$	$[U(6) \times Sp(4)]_{5_1} \times [U(6) \times Sp(4)]_{5_2}$	$Q^{(1\,2)}$	$(6,1;6,1)(+1;+1)$
		$S^{(1\,2)}$	$(\bar{6},1;1,4)(-1;0)$
		$U^{(1\,2)}$	$(1,4;\bar{6},1)(0;-1)$
$5_3 5_i$	$[U(4) \times U(2) \times SU(2) \times SU(2)]_{5_3}$	$P^{(3\,i)}$	$(1,2,1,1;6,1)(0,+1;+1)$
$i=1,2$	$\times [U(6) \times Sp(4)]_{5_i}$	$Q^{(3\,i)}$	$(4,1,1,1;6,1)(+1,0;+1)$
		$R^{(3\,i)}$	$(1,2,1,1;1,4)(0,-1;0)$
		$S^{(3\,i)}$	$(\bar{4},1,1,1;1,4)(-1,0;0)$
		$T^{(3\,i)}$	$(1,1,2,1;\bar{6},1)(0,0;-1)$
		$U^{(3\,i)}$	$(1,1,1,2;\bar{6},1)(0,0;-1)$

Table 2: The massless spectrum from the open string states. The indices k on the $\chi^{(s)}$ and $\psi^{(s)}$ fields are family indices, i.e. $k = 1, 2, 3$.

The twisted sector moduli fields arise from different twisted sectors and are associated with the fixed points of the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold. The twisted sectors are generated by the multiplications of elements of $\mathbb{Z}_3$ ($\{1, \theta \equiv \mathrm{diag}(e^{2\pi i/3}, e^{2\pi i/3}, e^{2\pi i/3}), \theta^2\}$), and the two $\mathbb{Z}_2$'s, generated by $R_1 = \mathrm{diag}(1, -1, -1)$ and $R_2 = \mathrm{diag}(-1, 1, -1)$, respectively. Each sector is specified by a certain number of fixed points (and/or fixed two-tori). However, not all the fixed points in these sectors are in one-to-one correspondence with the physical states, such as blowing-up moduli. Generally, in each sector only particular combinations of blowing-up modes that are invariant under the action of the remaining discrete rotations survive.

The blowing-up modes of the $\mathbb{Z}_3$ twisted sectors can be identified in the following way. The θ sector has $27 = 3\times 3\times 3$ fixed points, and twisted fields σ_i^J are associated with the i-th fixed point ($i = 1, 2, 3$) of the J-th two-torus T^2 ($J = 1, 2, 3$). The $\mathbb{Z}_3$ orientifold would have 27 blowing-up moduli fields of the form $\phi_{i,j,k} \equiv \sigma_i^1\sigma_j^2\sigma_k^3$, where $(i, j, k) = \{1, 2, 3\}$ run over the three fixed points on each torus. However, the invariance of the states under the two discrete $\mathbb{Z}_2$-rotations R_1 and R_2 implies that only nine combinations of the original blowing-up modes are physical. Those are (a) the blowing-up mode at the origin, (b) three blowing-up modes that are symmetric combinations of $\phi_{i,j,k}$ with one of the indices being 2 and the other two indices fixed to be 1 and its mirror image under $\mathbb{Z}_2$ twists, (c) three blowing-up modes with a symmetric combinations of $\phi_{i,j,k}$ with two of the indices being 2 and the other index fixed to be 1 and its three mirror images under $\mathbb{Z}_2$ twists and (d) two blowing-up modes which are symmetric and antisymmetric combinations of $\phi_{i,j,k}$ with $i = j = k = 2$ and its five images under $\mathbb{Z}_2$ twists.

The 16 blowing-up modes of each of the R_i twists are combined to generate 6 physical moduli that are also invariant under the $\mathbb{Z}_3$ action (cf. appendix B). The order 6 twists θR_i ($i = 1, 2, 3$) each have two blowing-up moduli ($R_3 \equiv R_1R_2 = (-1, -1, 0)$), which can be seen as follows, using θR_3 as an example. The θR_3 rotation is generated by $\mathrm{diag}(e^{\frac{-2\pi i}{6}}, e^{\frac{-2\pi i}{6}}, e^{\frac{2\pi i}{3}})$. The order-6 twists in both the first and second tori generate one fixed point (at the origin in each of the two-tori). The third torus has three fixed points due to the order-three twist. However, the R_1 and R_2 invariance selects out the two physical modes, $\rho_{1,1,1}$ and $\rho_{1,1,[2,3]} \equiv \frac{1}{\sqrt{2}}(\rho_{1,1,2} - \rho_{1,1,3})$ (the relative minus sign is a consequence of the torsion in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ sector), which combines the two states that are mirror pairs under R_1 and R_2.

3.1 Chern-Simons terms

Anomaly cancellation is ensured by the existence of the Chern-Simons (CS) term of the effective theory [17], which is of the form

$$I_{CS} \sim \sum_k \int d^4x \; C_k \wedge e^F \,, \tag{3.1}$$

where C_k are the 2-form Ramond-Ramond moduli fields arising from the k-th twisted sector and F is the field strength of the gauge bosons. The first order expansion in F gives the term

$$\sum_k \int d^4x \; C_k \wedge \mathrm{Tr}\left(\gamma_k \lambda^{(0)}\right) F \,, \tag{3.2}$$

	$\mathrm{Tr}\left(\gamma_g \lambda^{(0)}\right)$	$\mathrm{Tr}\left((\gamma_g)^{-1}(\lambda^{(0)})^2\right)$	
γ_g	U(6)	U(6)	Sp(4)
$\gamma_{\mathbb{Z}_3}$	$2i\sqrt{3}\,\mathrm{Tr}\,B_1$	$-2\,\mathrm{Tr}\,B_1^2$	$4\,\mathrm{Tr}\,(B_2^2 + S_2 S_1)$
$\gamma_{\mathbb{Z}_3}^2$	$-2i\sqrt{3}\,\mathrm{Tr}\,B_1$	$-2\,\mathrm{Tr}\,B_1^2$	$4\,\mathrm{Tr}\,(B_2^2 + S_2 S_1)$

Table 3: Contributions of the different gauge groups to the traces relevant for anomaly cancellation, FI-terms and gauge coupling corrections. B_1 represents a U(6) generator, while B_2 and the S_i generate the Sp(4) (see the detailed explanation after eq. (2.22)). All the other traces, e.g. those involving $\mathbb{Z}_2$ twists γ_{R_i}, vanish.

where the coefficient $\mathrm{Tr}(\gamma_k \lambda^{(0)})$ is associated to the orbifold action and the Chan-Paton matrix of the gauge bosons. The second order expansion gives the coupling between the twisted sector R-R fields and $F\tilde{F}$, with the prefactor $\mathrm{Tr}((\gamma_k)^{-1}{\lambda^{(0)}}^2)$. These two couplings are responsible for the cancellation of field theoretical U(1) anomalies [16].

The particular combinations of R-R twisted sector fields that are responsible for U(1) anomaly cancellations require careful study. For example, the explicit trace calculation for the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold without Wilson line (table 3) reveals that $\mathrm{Tr}(\gamma_k \lambda^{(0)})$ vanishes for all the twists except those of $\mathbb{Z}_3$. It implies that the R-R twisted moduli that are responsible for anomaly cancellation are from the $\mathbb{Z}_3$ twisted sectors only. For the U(1)'s arising from the D9-branes, which extend over all six compactified dimensions, all the physical R-R field are involved. Namely, the R-R field for anomaly cancellation in the D9-brane sector B^9 takes the form

$$B^9 \propto \sum_{i,j,k} \psi_{i,j,k}\,, \tag{3.3}$$

where $\psi_{i,j,k}$ are the R-R partner of the twisted sector moduli $\phi_{i,j,k}$ of $\mathbb{Z}_3$. For the $\mathrm{D5}_3$-branes sitting at the origin, extending over the i-th complex coordinate, the relevant R-R field for anomaly cancellation B^{5_3} takes the form

$$B^{5_3} \propto \sum_{i} \ \psi_{1,1,i}\,. \tag{3.4}$$

B_{5_1} and B_{5_2} have similar forms. The situation in the model with Wilson line is more complicated and will be discussed later.

3.2 Anomaly cancellation

The U(1) mixed anomalies in the effective theory are cancelled by the exchange of twisted sector R-R fields between the gauge bosons, as discussed in [16]. The amplitude for this process is given by the expression

$$A_{lm}^{\alpha\beta} = \frac{i}{|P|} \sum_k C_k^{\alpha\beta}(v)\,\mathrm{Tr}\left(\gamma_k^\alpha \lambda_l^{(0)}\right) \mathrm{Tr}\left(\left(\gamma_k^\beta\right)^{-1}\left(\lambda_m^{(0)}\right)^2\right), \tag{3.5}$$

JHEP04(2000)004

where $\alpha,\ \beta = 9,\ 5_i$ denote the brane sectors from which the U(1) group and the non-abelian group arise; P is the order of the orbifold group, which for the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold takes the value $2 \times (2 \times 2 \times 3) = 24$, and k runs over all the twisted sectors. The factor $C_k^{\alpha\beta}$ arises from string tadpole calculations. In the case of $\alpha = \beta$,

$$C_k^{\alpha\beta}(v) = (-1)^k\, \Pi_{a=1}^3\, 2\sin(\pi k v_a)\,. \tag{3.6}$$

$\mathbb{Z}_3$ action gives $C_1^{\alpha\beta}(v) = -C_2^{\alpha\beta}(v) = -3\sqrt{3}$. If $\alpha = 9$ and $\beta = 5_i$,

$$C_k^{95_i}(v) = (-1)^k 2\sin(\pi k v_i)\,, \tag{3.7}$$

which gives $C_1^{95_i}(v) = -C_2^{95_i}(v) = -\sqrt{3}$ for $\mathbb{Z}_3$. If $\alpha = 5_i$ and $\beta = 5_j$,

$$C_k^{5_i 5_j}(v) = (-1)^k 2\sin(\pi k v_a)\,, \quad \text{where } a \neq i \neq j\,. \tag{3.8}$$

Hence, the order 3 twists of the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold give $C_1^{5_i 5_j}(v) = -C_2^{5_i 5_j}(v) = -\sqrt{3}$.

When a background Wilson line is introduced into the world-volume of the D9-branes, the fixed points of the orbifold action generally split into different sets, each of which feels different gauge monodromy. The orbifold action on the Chan-Paton matrices is modified accordingly at each fixed point, and the total amplitude A_{lm} of the R-R twisted moduli exchange has to be averaged over all the fixed points of the orbifold action [16],

$$A_{lm}^{\alpha\beta} = \frac{i}{|P|}\frac{1}{F}\sum_k \sum_f\ C_k^{\alpha\beta}(v)\,\mathrm{Tr}\left(\gamma_f^\alpha \lambda_l^{(0)}\right)\mathrm{Tr}\left(\left(\gamma_f^\beta\right)^{-1}\left(\lambda_m^{(0)}\right)^2\right), \tag{3.9}$$

where F is the total number of fixed points, f runs over all the fixed points of the k-th twisted sector and γ_f is the modified orbifold action at each fixed point due to Wilson line actions. For example, in a $\mathbb{Z}_3$ orientifold model with one Wilson line, the 27 fixed points of the $\mathbb{Z}_3$ action split into 3 sets, each of them feeling different monodromy, $\gamma_{\mathbb{Z}_3}$, $\gamma_{\mathbb{Z}_3}\gamma_W$ and $\gamma_{\mathbb{Z}_3}\gamma_W^2$, respectively. In the present model, the situation is even more complicated as we will discuss later.

(i) Anomaly cancellation of the $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ model without Wilson lines. The four U(1)'s from each of the U(6) groups are anomalous. The mixed triangular anomalies between the four U(1)'s and the SU(6) and Sp(4) groups from each brane sector can be easily calculated from the particle spectrum [11]

$$\tilde{A} = \begin{pmatrix} 9 & -9 & 3 & -3 & 3 & -3 & 3 & -3 \\ 3 & -3 & 9 & -9 & 3 & -3 & 3 & -3 \\ 3 & -3 & 3 & -3 & 9 & -9 & 3 & -3 \\ 3 & -3 & 3 & -3 & 3 & -3 & 9 & -9 \end{pmatrix}, \tag{3.10}$$

where the row l labels the U(1) from the D9- and D5$_i$-brane sectors and the column m labels the SU(6) and Sp(4) groups. As can be seen from table 3, only $\mathbb{Z}_3$ twisted moduli contribute to the anomaly cancellation process and eq. (3.5) indeed cancels the anomalies exactly. For example,

$$A_{1\,1} = \frac{i}{24} \times \left(-3\sqrt{3}\right) \times 2 \times \left[2i\sqrt{3}\,\mathrm{Tr}\, B_1^9 \left(-2\,\mathrm{Tr}\left(B_1^9\right)^2\right)\right] = -9\,, \qquad (3.11)$$

where we used the standard SU(N) normalization $\mathrm{Tr}\, B_1^2 = 1/2$ for the SU(6) generator, while the U(1) generators are unnormalized. (We concentrate on the cancellation of mixed anomalies between the U(1) groups and the non-abelian symmetries, the U(1)^3 anomaly and the mixed anomalies between U(1) and gravity are cancelled in a similar way [16].)

(ii) Anomaly cancellation of the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ model with one Wilson line. The field theoretical anomalies between U(1)'s and the non-abelian groups in this model can be summarized in the following matrix:

$$\tilde{A} = \begin{pmatrix} 6 & 0 & 0 & -6 & 2 & -2 & 2 & -2 & 2 & 0 & 0 & -2 \\ 0 & 1 & -1 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 3 & 3 & -3 & -3 & 9 & -9 & 3 & -3 & 3 & 3 & -3 & -3 \\ 3 & 3 & -3 & -3 & 3 & -3 & 9 & -9 & 3 & 3 & -3 & -3 \\ 2 & 0 & 0 & -2 & 2 & -2 & 2 & -2 & 6 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 1 & -1 & 0 \end{pmatrix}, \qquad (3.12)$$

where the row l denotes U(1) factors from the U(4) and U(2) of the D9-brane sector, the U(6)'s of the D5$_{1,2}$-brane sector, and the U(4) and U(2) of the D5$_3$-brane sector. The columns count the non-abelian groups, the order follows table 2.

The introduction of the Wilson line implies a more involved pattern of anomaly cancellation. In the following we separately address the anomaly cancellation for different sets of "anomalous" U(1)'s.

(1) The anomalies involving the U(1) $\subset$ U(4)**.** As shown in table 4, non-zero contributions to anomaly cancellation, as specified by eq. (3.9) come *only* from the $\mathbb{Z}_3$ twisted sectors, e.g. in the case of the mixed anomaly U(1) $\times$ SU(4)^2, where both of the gauge groups are from the 99 sector, eq. (3.9) yields

$$A_{1\,1} = \frac{i}{24} \times \frac{1}{3} \times \left(-3\sqrt{3}\right) \times 2 \times \left[3 \times 2i\sqrt{3}\,\mathrm{Tr}\, \tilde{B}_2^9 \left(-2\,\mathrm{Tr}\left(\tilde{B}_2^9\right)^2\right)\right] = -6\,, \quad (3.13)$$

where we have used $C_1^{99} = -C_2^{99} = -3\sqrt{3}$ for the $\mathbb{Z}_3$ twist, and $\mathrm{Tr}\,\tilde{B}_2 = 4$ for the unnormalized U(1) factor and $\mathrm{Tr}\,\tilde{B}_2^2 = 1/2$ for the properly normalized SU(4)

	$\mathrm{Tr}\left(\gamma_g\lambda^{(0)}\right)$		$\mathrm{Tr}\left((\gamma_g)^{-1}(\lambda^{(0)})^2\right)$			
γ_g	U(4)	U(2)	U(4)	U(2)	SU(2)	SU(2)
γ_{Z_3}	$2i\sqrt{3}\mathrm{Tr}\tilde{B}_2$	$2i\sqrt{3}\mathrm{Tr}\tilde{B}_1$	$-2\mathrm{Tr}\tilde{B}_2^2$	$-2\mathrm{Tr}\tilde{B}_1^2$	$4\left(W_1^2+W_2W_3\right)$	$4\left(Z_1^2+Z_2Z_3\right)$
$\gamma_{Z_3}^2$	$-2i\sqrt{3}\mathrm{Tr}\tilde{B}_2$	$-2i\sqrt{3}\mathrm{Tr}\tilde{B}_1$	$-2\mathrm{Tr}\tilde{B}_2^2$	$-2\mathrm{Tr}\tilde{B}_1^2$	$4\left(W_1^2+W_2W_3\right)$	$4\left(Z_1^2+Z_2Z_3\right)$
$\gamma_{Z_3}\gamma_W$	$2i\sqrt{3}\mathrm{Tr}\tilde{B}_2$	$-i\sqrt{3}\mathrm{Tr}\tilde{B}_1$	$-2\mathrm{Tr}\tilde{B}_2^2$	$\mathrm{Tr}\tilde{B}_1^2$	$-2\left(W_1^2+W_2W_3\right)$	$4\left(Z_1^2+Z_2Z_3\right)$
$\gamma_{Z_3}^2\gamma_W$	$-2i\sqrt{3}\mathrm{Tr}\tilde{B}_2$	$i\sqrt{3}\mathrm{Tr}\tilde{B}_1$	$-2\mathrm{Tr}\tilde{B}_2^2$	$\mathrm{Tr}\tilde{B}_1^2$	$-2\left(W_1^2+W_2W_3\right)$	$4\left(Z_1^2+Z_2Z_3\right)$
$\gamma_{R_3}\gamma_W$	0	0	0	$-2\sqrt{3}\mathrm{Tr}\tilde{B}_1^2$	$-2\sqrt{3}\left(W_1^2+W_2W_3\right)$	0
$\gamma_{Z_3}\gamma_{R_3}\gamma_W$	0	$-3i\mathrm{Tr}\tilde{B}_1$	0	$\sqrt{3}\mathrm{Tr}\tilde{B}_1^2$	$-2\sqrt{3}\left(W_1^2+W_2W_3\right)$	0
$\left(\gamma_{Z_3}\gamma_{R_3}\right)^5\gamma_W$	0	$3i\mathrm{Tr}\tilde{B}_1$	0	$\sqrt{3}\mathrm{Tr}\tilde{B}_1^2$	$-2\sqrt{3}\left(W_1^2+W_2W_3\right)$	0
$\gamma_{Z_3}\gamma_W^2$	$2i\sqrt{3}\mathrm{Tr}\tilde{B}_2$	$-i\sqrt{3}\mathrm{Tr}\tilde{B}_1$	$-2\mathrm{Tr}\tilde{B}_2^2$	$\mathrm{Tr}\tilde{B}_1^2$	$-2\left(W_1^2+W_2W_3\right)$	$4\left(Z_1^2+Z_2Z_3\right)$
$\gamma_{Z_3}^2\gamma_W^2$	$-2i\sqrt{3}\mathrm{Tr}\tilde{B}_2$	$i\sqrt{3}\mathrm{Tr}\tilde{B}_1$	$-2\mathrm{Tr}\tilde{B}_2^2$	$\mathrm{Tr}\tilde{B}_1^2$	$-2\left(W_1^2+W_2W_3\right)$	$4\left(Z_1^2+Z_2Z_3\right)$
$\gamma_{R_3}\gamma_W^2$	0	0	0	$2\sqrt{3}\mathrm{Tr}\tilde{B}_1^2$	$2\sqrt{3}\left(W_1^2+W_2W_3\right)$	0
$\gamma_{Z_3}\gamma_{R_3}\gamma_W^2$	0	$3i\mathrm{Tr}\tilde{B}_1$	0	$-\sqrt{3}\mathrm{Tr}\tilde{B}_1^2$	$2\sqrt{3}\left(W_1^2+W_2W_3\right)$	0
$\left(\gamma_{Z_3}\gamma_{R_3}\right)^5\gamma_W^2$	0	$-3i\mathrm{Tr}\tilde{B}_1$	0	$-\sqrt{3}\mathrm{Tr}\tilde{B}_1^2$	$2\sqrt{3}\left(W_1^2+W_2W_3\right)$	0

Table 4: Contributions of the different gauge groups in the D9- and D5$_3$-brane sectors to the traces relevant for anomaly cancellation, FI-terms and gauge coupling corrections for the model with the Wilson line. $\tilde{B}_1$ and $\tilde{B}_2$ are the U(2) and U(4) generators and the W_i and Z_i form the SU(2) generators. In the D5$_{1,2}$-brane sectors the traces remain unchanged, i.e. they can be found in table 3.

generator. Similarly, using the results in table 4, we find that all the entries $\tilde{A}_{1\,2}-\tilde{A}_{1\,4}$ are exactly cancelled by A_{lm}. However, for the anomalies $\mathrm{U}(1)\times G_i^2$, where G_i comes from the D5$_1$- or D5$_2$-branes, the situation is different. Since we assume that the D5$_1$- and D5$_2$-branes sit at the origin of the third two-torus (as well as the origin of the second and first two-tori), they do not feel the action of the Wilson line, which is acting on the third complex plane. Therefore, the result from eq. (3.9) is simply

$$A_{1\,5}=\frac{i}{24}\times\left(-\sqrt{3}\right)\times 2\times 2i\sqrt{3}\,\mathrm{Tr}\,\tilde{B}_2^9\left(-2\,\mathrm{Tr}\left(B_1^{5_1}\right)^2\right)=-2\,, \tag{3.14}$$

where we have $C_1^{95_i}=-C_1^{95_i}=-\sqrt{3}$ as in the case without Wilson line, and B_1 is the generator for the U(6) group in the D5$_1$-brane sector. Similarly, $\tilde{A}_{1\,6}-\tilde{A}_{1\,8}$ involving the gauge groups Sp(4) (in D5$_1$-brane sector), SU(6) and Sp(4) (in D5$_2$-brane sector) are cancelled. $\tilde{A}_{1\,9}-\tilde{A}_{1\,12}$ involve G_i from the D5$_3$-brane sector, where we again have to take the Wilson line action into account. The cancellation works in the same way as in the case of $\tilde{A}_{1\,1}-\tilde{A}_{1\,4}$. For example,

$$A_{1\,9}=\frac{i}{24}\times\frac{1}{3}\times\left(-\sqrt{3}\right)\times 2\times\left[3\times 2i\sqrt{3}\,\mathrm{Tr}\,\tilde{B}_2^9\left(-2\,\mathrm{Tr}\left(\tilde{B}_2^{5_3}\right)^2\right)\right]=-2\,. \tag{3.15}$$

(2) Anomalies involving $\mathrm{U}(1)\subset\mathrm{U}(2)$. The relevant traces in table 4 show that in the presence of the background Wilson line, not only the $\mathbb{Z}_3$ twisted sectors, but also the order 6 twists generated by $\mathbb{Z}_3 \times R_3$ contribute to the anomaly cancellation. The latter, however, comes with different C_k factors. Since the order 6 twist is generated by the action $\mathrm{diag}(e^{-2\pi i/6}, e^{-2\pi i/6}, e^{2\pi i/3})$, $C_1^{99} = -C_5^{99} = \sqrt{3}$. Hence, $A_{2\,2}$ takes the form

$$A_{2\,2} = \frac{i}{24} \times \Big\{ \frac{1}{3} \times \big(-3\sqrt{3} \big) \times 2 \times \Big[2i\sqrt{3}\,\mathrm{Tr}\,\tilde{B}_1^9 \Big(-2\,\mathrm{Tr}\,\big(\tilde{B}_1^9\big)^2 \Big) + \\ + 2 \times 2i\sqrt{3}\,\mathrm{Tr}\,\tilde{B}_1^9 \Big(-\mathrm{Tr}\,\big(\tilde{B}_1^9\big)^2 \Big) \Big] + \\ + \frac{1}{3} \times \big(\sqrt{3} \big) \times 2 \times \Big[2 \times (-3)\,\mathrm{Tr}\,\tilde{B}_1^9 \Big(i\sqrt{3}\,\mathrm{Tr}\,\big(\tilde{B}_1^9\big)^2 \Big) \Big] \Big\} = -1\,. \quad (3.16)$$

One sees that among the various contributions to $A_{2\,1}$, in the $\mathbb{Z}_3$ twisted sectors the contribution from fixed points which do not feel the Wilson line cancel those from fixed points which do feel the Wilson line; while in $\mathbb{Z}_6$ twisted sectors, the contributions from fixed points that feel the Wilson line action cancel between each other. As a net result, $A_{2\,1} = 0$. $A_{2\,3}$ and $A_{2\,4}$ cancel their counterparts from $\tilde{A}$ as well. In cancelling $\tilde{A}_{2\,5}$–$\tilde{A}_{2\,8}$ in which the non-abelian group arises from the 5_1 or 5_2 sector, the Wilson line action does not have any effect, thus $\tilde{A}_{2\,5}$ is cancelled by

$$A_{2\,5} = \frac{i}{24} \times \big(-\sqrt{3} \big) \times 2 \times 2i\sqrt{3}\,\mathrm{Tr}\,\tilde{B}_1^9 \left(-2\,\mathrm{Tr}\,\big(B_1^{5_1}\big)^2 \right) = -1\,. \quad (3.17)$$

When G_i comes from the D5_3-brane sector, the effects from the $\mathbb{Z}_6$ twists again have to be taken into account. The net result is that the contribution from the $\mathbb{Z}_3$ twists cancels those from the $\mathbb{Z}_6$ twists in such a way that $A_{2\,9} - A_{2\,12} = 0$.

(3) Anomalies involving $\mathrm{U}(1) \subset \mathrm{U}(6)$**.** Since the U(6) group arises from the D5_1- or D5_2-brane sectors, where the Wilson line acts trivially, the cancellation of anomalies of this type is the same as in the original model without Wilson line.

We have also checked explicitly all the $\mathrm{U}(1)^3$ anomalies in the model, and confirmed that they are exactly cancelled by the generalized GS mechanism.

4. Fayet-Iliopoulos terms and gauge coupling corrections

4.1 Fayet-Iliopoulos terms

The supersymmetric completion of the term (3.2) generates the Fayet-Iliopoulos (FI) terms for the associated anomalous U(1)'s.[4] In the case of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold, the FI term of the $\mathrm{U}(1)_A$ in the D9-brane sector takes the form

$$\xi_{FI}^9 \sim \int d^4x\ \mathrm{Tr}\left(\gamma_{\mathbb{Z}_3}\lambda^{(0)}\right) \sum_{i,j,k=1}^{3} \phi_{i,j,k}\,, \quad (4.1)$$

[4]Note that there could be sigma-model anomaly corrections to the FI-terms [4], proportional to the untwisted sector moduli. (See also ref. [20].)

where $\phi_{i,j,k}$ are the NS-NS sector moduli arising from the $\mathbb{Z}_3$ twisted sectors, since, as we argued earlier only $\mathbb{Z}_3$ twisted sectors contribute to the anomaly cancellation process. (Notice that we are summing over only half of the twisted sectors.) The FI terms of the anomalous U(1) from the 5_1 branes involve the NS-NS moduli $\phi_{i,1,1}$, those from 5_2 branes involve $\phi_{1,i,1}$, etc. In the following, we discuss the FI terms when a background Wilson line is added.

(a) The FI term of the U(1)'s from D9-branes. For the anomalous $U(1) \subset U(4)$,

$$\begin{aligned}
\xi^9_{FI,1} &\sim \sum_{i=1}^{3}\sum_{j=1}^{3}\left[\operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\lambda^{(0)}\right)\phi_{i,j,1} + \operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W\lambda^{(0)}\right)\phi_{i,j,2} + \operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W^2\lambda^{(0)}\right)\phi_{i,j,3}\right] \\
&= \left(2i\sqrt{3}\right)\operatorname{Tr}\tilde{B}^9_2 \sum_{i,j,k=1}^{3}\left(\phi_{i,j,k}\right),
\end{aligned} \tag{4.2}$$

where $\phi_{i,j,k}$ are moduli fields from $\mathbb{Z}_3$ twisted sectors and the overall normalization factor has been suppressed. Note that, although the blowing-up modes $\phi_{i,j,k}$ form a convenient basis, they are not physical moduli of $\mathbb{Z}_3$. As discussed earlier, only a specific combination of nine blowing-up modes is physical.

In the case of $U(1) \subset U(2)$, the order 6 twists also contribute,

$$\begin{aligned}
\xi^9_{FI,2} &\sim \sum_{i=1}^{3}\sum_{j=1}^{3}\left[\operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\lambda^{(0)}\right)\phi_{i,j,1} + \operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W\lambda^{(0)}\right)\phi_{i,j,2} + \operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_W^2\lambda^{(0)}\right)\phi_{i,j,3}\right] + \\
&\quad + \left[\operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_{R3}\lambda^{(0)}\right)\rho_{1,1,1} + \operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_{R3}\gamma_W\lambda^{(0)}\right)\rho_{1,1,2} + \operatorname{Tr}\left(\gamma_{\mathbb{Z}_3}\gamma_{R3}\gamma_W^2\lambda^{(0)}\right)\rho_{1,1,3}\right] \\
&= \left(i\sqrt{3}\right)\operatorname{Tr}\tilde{B}^9_1 \sum_{i=1}^{3}\sum_{j=1}^{3}(2\phi_{i,j,1} - \phi_{i,j,2} - \phi_{i,j,3}) + (-3i)\operatorname{Tr}\tilde{B}^9_1(\rho_{1,1,2} - \rho_{1,1,3})\,.
\end{aligned} \tag{4.3}$$

(b) The FI terms of the U(1)'s from D5$_3$-branes. Since the 5_3 sector is subject to the Wilson line action, the basic forms of the FI terms are the same as in the D9-brane sector. However, since 5_3 branes sit at the origin of the 1st and 2nd complex tori, only those moduli with $i = j = 1$ are involved. Hence,

$$\xi^{5_3}_{FI,1} \sim \left(2i\sqrt{3}\right)\operatorname{Tr}\tilde{B}^{5_3}_2(\phi_{1,1,1} + \phi_{1,1,2} + \phi_{1,1,3})\,, \tag{4.4}$$

is the FI term for the $U(1) \subset U(4)$. On the other hand,

$$\xi^{5_3}_{FI,2} \sim \left(i\sqrt{3}\right)\operatorname{Tr}\tilde{B}^{5_3}_1(2\phi_{1,1,1} - \phi_{1,1,2} - \phi_{1,1,3}) + (-3i)\operatorname{Tr}\tilde{B}^{5_3}_1(\rho_{1,1,2} - \rho_{1,1,3})\,, \tag{4.5}$$

gives the FI term for the $U(1) \subset U(2)$.

(c) The FI terms of the U(1)'s that arise from D5$_1$- and D5$_2$-branes. As suggested by the anomaly cancellations, only moduli associated to the $\mathbb{Z}_3$ twists are relevant. Hence, the FI term for the U(1) in the 5_1 brane sector is

$$\xi_{FI}^{5_1} \sim \left(2i\sqrt{3}\right) \mathrm{Tr}\, \tilde{B}_1^{5_1} (\phi_{1,1,1} + \phi_{2,1,1} + \phi_{3,1,1})\,, \tag{4.6}$$

and for the U(1) from D5$_2$-branes,

$$\xi_{FI}^{5_2} \sim \left(2i\sqrt{3}\right) \mathrm{Tr}\, \tilde{B}_1^{5_2} (\phi_{1,1,1} + \phi_{1,2,1} + \phi_{1,3,1})\,. \tag{4.7}$$

4.2 Gauge coupling corrections

The gauge kinetic functions receive corrections from the twisted sector blowing-up modes, in a mechanism related to the Chern-Simons term [4, 12, 18, 19]; they have their origin in the second-order corrections, and the proposed form of the corrections [4] was confirmed by recent explicit string calculations [18]. The holomorphic gauge coupling function is of the form

$$f = S + \delta f(R)\,, \tag{4.8}$$

where S is the (untwisted sector) dilaton for D9-brane sector or the untwisted toroidal modulus T_i for D5$_i$-brane sectors. For a $\mathbb{Z}_N$ orbifold, the correction can be expressed as

$$\delta f(R) \sim \sum_k \mathrm{Tr}\left(\gamma_{\theta^k} \lambda^{(0)2}\right) \sum_{l,m,n} \phi_{l,m,n}\,, \tag{4.9}$$

where the summation is over half of the twisted sectors, and $\phi_{l,m,n}$ are the NS-NS twisted sector moduli associated with the particular twist. (Again an overall normalization factor is suppressed.) In the case of the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ model, δf receives non-trivial contributions from $\mathbb{Z}_3$ twists only (cf. table 3), such that

$$\delta f_1^9 \sim -2\, \mathrm{Tr}\left(B_2^9\right)^2 \sum_{i,j,k=1}^{3} (\phi_{i,j,k}) \tag{4.10}$$

and

$$\delta f_2^9 \sim 4\, \mathrm{Tr}\left(\left(B_2^9\right)^2 + S_2^9 S_1^9\right) \sum_{i,j,k}^{3} (\phi_{i,j,k}) \tag{4.11}$$

for the SU(6) and Sp(4) groups in the D9-brane sector. The corrections take a similar form for gauge groups arising from the D5$_i$-brane sector, except that the summations over i, j, k are replaced by fixed values $j = k = 1$, $k = i = 1$ and $i = j = 1$, respectively.

With the Wilson line, in the D9-brane sector, table 4 shows that the gauge coupling of SU(4) receives non-zero corrections, as specified by eq. (4.9), only from $\mathbb{Z}_3$ twists. It takes the form

$$\delta f_1^9 \sim -2\, \mathrm{Tr}\left(\tilde{B}_2^9\right)^2 \sum_{i,j,k=1}^{3} (\phi_{i,j,k})\,, \tag{4.12}$$

while the gauge coupling correction for SU(2)($\subset$ U(2)), gets contributions from order 3, *and* order 6 twisted sectors. It is of the form

$$\delta f_2^9 \sim -\,\mathrm{Tr}\left(\tilde{B}_1^9\right)^2 \sum_{i,j}^{3}(2\phi_{i,j,1}-\phi_{i,j,2}-\phi_{i,j,3}) + \sqrt{3}\,\mathrm{Tr}\left(\tilde{B}_1^9\right)^2(\rho_{1,1,2}-\rho_{1,1,3})\,. \qquad (4.13)$$

Notice that from table 4, the order 2 twist R_3 appears to have a contribution to eq. (4.9) with the relevant traces being non-zero. However, since R_3 acts trivially on the third complex torus, the Wilson line action does not differentiate the moduli of the R_3 twist. Hence, the total contribution from the R_3 sector to δf vanishes after summing over γ_W^k.

Similarly, for the first SU(2) $\equiv$ Sp(2) group in the D9-brane sector one obtains

$$\begin{aligned}\delta f_3^9 \sim\; & 2\left(\left(W_1^9\right)^2 + W_2^9 W_3^9\right)\sum_{i,j}^{3}(2\phi_{i,j,1}-\phi_{i,j,2}-\phi_{i,j,3}) - \\ & -2\sqrt{3}\left(\left(W_1^9\right)^2 + W_2^9 W_3^9\right)(\rho_{1,1,2}-\rho_{1,1,3})\,. \qquad (4.14)\end{aligned}$$

And the second SU(2) receives the following correction to its gauge kinetic function:

$$\delta f_4^9 \sim 4\left(\left(Z_1^9\right)^2 + Z_2^9 Z_3^9\right)\sum_{i,j,k}^{3}(\phi_{i,j,k})\,. \qquad (4.15)$$

The corrections to the gauge couplings in the D5_3-brane sector are very similar to those in the D9-brane sector, except that the summation over i,j is changed to a fixed value $i=j=1$.

In the D5_1 sector, the correction to the gauge coupling of SU(6) is simply

$$\delta f_1^{5_1} \sim -2\,\mathrm{Tr}\left(B_1^{5_1}\right)^2 \sum_{i=1}^{3}(\phi_{i,1,1})\,, \qquad (4.16)$$

and that of Sp(4) is

$$\delta f_2^{5_1} \sim 4\,\mathrm{Tr}\left((B_2^{5_1})^2 + S_2^{5_1}S_1^{5_1}\right)\sum_{i=1}^{3}(\phi_{i,1,1})\,. \qquad (4.17)$$

The corrections to the gauge functions in the D5_2-brane sector are similar to those in the D5_1 sector, except that the moduli fields which are involved are $\phi_{1,i,1}$.

5. Conclusions

We have presented an explicit construction of a $N=1$ supersymmetric, three family, type-IIB $\mathbb{Z}_3\times\mathbb{Z}_2\times\mathbb{Z}_2$ orientifold with a Wilson line that does not commute with the orbifold group. One of the motivations for this construction was to find a model with a gauge group structure that is close to that of the standard model and thus provide an example of a model with potentially quasi-realistic features. Unfortunately, the only examples that are free of non-abelian anomalies and are consistent with tadpole cancellation, while still containing the SM gauge group as a subgroup, have a gauge group structure of the type $\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2)(\times \mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times (\mathrm{U}(6) \times \mathrm{Sp}(4))^2)$, and thus its "observable" sector gauge structure is still much larger than that of the SM. (While our study builds on an earlier work [11], we found a different gauge group structure and massless spectrum).

The gauge group $\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ can be easily understood in the T-dual picture, in which the Wilson line action on the $\mathrm{D5}_3$-branes is dual to splitting the 32 $\mathrm{D5}_3$-branes and placing them at various fixed points, while respecting the orbifold symmetries. In the T-dual picture, a set of 20 $\mathrm{D5}_3$-branes is placed at the origin, which gives rise to the gauge group $\mathrm{U}(4) \times \mathrm{Sp}(2)$ when the $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold actions are imposed. A set of 6 $\mathrm{D5}_3$-branes is placed at one of the two fixed points of $\mathbb{Z}_3$ in the third complex two-torus which are away from the origin. Due to the two $\mathbb{Z}_2$ actions, another set of 6 $\mathrm{D5}_3$-branes needs to be placed at the other fixed point as the mirror image of the first set of 6. These two sets of 6 $\mathrm{D5}_3$-branes yield only one set of physical states and give rise to the gauge group $\mathrm{U}(2) \times \mathrm{Sp}(2)$ under the additional $\mathbb{Z}_3$ orbifold action.

In addition to obtaining the gauge group structure, the massless spectrum and the trilinear superpotential, we carried out the explicit calculation of the abelian anomaly cancellations, employing a generalization of the Green-Schwarz mechanism (as proposed in ref. [16] and confirmed in ref. [18]). In particular we emphasized subtleties associated with "anomalous" U(1)'s of U(4) and U(2) group factors, which are due to the non-trivial role non-abelian Wilson lines are playing in the anomaly cancellation. In addition, we also calculated the FI-terms and the gauge coupling corrections due to the blowing-up modes, thus setting a stage for further investigations of the physics implications of this type of models.

The techniques we have employed in order to obtain a specific anomaly free three family model can be easily applied to the study of a broader class of type-IIB orientifolds, i.e. models with other orbifold groups, and with a more general form of the non-abelian Wilson lines. In addition, a systematic formulation of the consistency constraints for this class of string solutions is clearly an important question, and deserves further study.

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JHEP04(2000)004

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A. The matter fields

99-sector: like for the gauge fields, the Chan-Paton matrices of the matter states have to be invariant under the action of the orientifold group. Since the world-sheet states of the matter fields are not invariant under the orientifold action, eqs. (2.2) and (2.3) now imply that

$$\lambda^{(i)} = -\gamma_{\Omega,9}\lambda^{(i)\,\mathrm{T}}\gamma_{\Omega,9}^{-1} \qquad \text{and} \qquad \lambda^{(i)} = e^{2i\pi v_i}\gamma_{g,9}\lambda^{(i)}\gamma_{g,9}^{-1}\,. \tag{A.1}$$

5_i5_i-sector: similarly, if open strings begin and end on D5_i-branes, the Chan-Paton matrices have to obey the following constraints:

$$\lambda^{(i)} = -\gamma_{\Omega,5}\lambda^{(i)\,\mathrm{T}}\gamma_{\Omega,5}^{-1}\,, \qquad \lambda^{(i)} = e^{2i\pi v_i}\gamma_{g,5_i}\lambda^{(i)}\gamma_{g,5_i}^{-1}\,, \tag{A.2}$$

$$\lambda^{(j)} = +\gamma_{\Omega,5}\lambda^{(j)\,\mathrm{T}}\gamma_{\Omega,5}^{-1}\,, \qquad \lambda^{(j)} = e^{2i\pi v_j}\gamma_{g,5_i}\lambda^{(j)}\gamma_{g,5_i}^{-1} \quad \text{for } i \neq j\,. \tag{A.3}$$

The sign change in the world-sheet parity projection in the last line stems from the DD boundary conditions in the $j \neq i$ directions transverse to the D5_i-branes.

95_i-sectors: further, one can have mixed states where the open strings begin and end on different branes. In the 95_i case, the coordinates obey mixed DN boundary conditions, i.e. they have half-integer modings. The states can be written as $|s_j, s_k, ab\rangle\,\lambda_{ab}$, $j,k \neq i$, with helicities $s_j, s_k = \pm 1/2$, and the Chan-Paton index a (b) lies on a D5-brane (D9-brane). Due to the GSO projection one has $s_j = s_k$. The orientifold projections now imply

$$\lambda = e^{2\pi i(v_j s_j + v_k s_k)}\gamma_{g,5_i}\lambda\gamma_{g,9}^{-1}\,. \tag{A.4}$$

The world-sheet parity operation Ω only relates 95_i and 5_i9 sectors and does not yield any additional constraints.

5_j5_i-sectors: finally, the matter states in mixed 5_j5_i-sectors are determined by

$$\lambda = e^{2\pi i(v_i s_i + v_j s_j)}\gamma_{g,5_i}\lambda\gamma_{g,5_j}^{-1}\,. \tag{A.5}$$

As an example, consider the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ orientifold discussed in the main text. In the 99-sector the matter Chan-Paton matrices have to satisfy

$$\lambda^{(1)} = e^{2i\pi/3}\gamma_{\mathbb{Z}_3,9}\lambda^{(1)}\gamma_{\mathbb{Z}_3,9}^{-1}\,, \quad \lambda^{(1)} = +\gamma_{R_1,9}\lambda^{(1)}\gamma_{R_1,9}^{-1}\,, \quad \lambda^{(1)} = -\gamma_{R_2,9}\lambda^{(1)}\gamma_{R_2,9}^{-1}\,, \tag{A.6}$$

$$\lambda^{(2)} = e^{2i\pi/3}\gamma_{\mathbb{Z}_3,9}\lambda^{(2)}\gamma_{\mathbb{Z}_3,9}^{-1}\,, \quad \lambda^{(2)} = -\gamma_{R_1,9}\lambda^{(2)}\gamma_{R_1,9}^{-1}\,, \quad \lambda^{(2)} = +\gamma_{R_2,9}\lambda^{(2)}\gamma_{R_2,9}^{-1}\,, \tag{A.7}$$

$$\lambda^{(3)} = e^{2i\pi/3}\gamma_{\mathbb{Z}_3,9}\lambda^{(3)}\gamma_{\mathbb{Z}_3,9}^{-1}\,, \quad \lambda^{(3)} = -\gamma_{R_1,9}\lambda^{(3)}\gamma_{R_1,9}^{-1}\,, \quad \lambda^{(3)} = -\gamma_{R_2,9}\lambda^{(3)}\gamma_{R_2,9}^{-1}\,. \tag{A.8}$$

JHEP04(2000)004

Imposing these constraints yields, e.g. for $\lambda^{(1)}$

$$\lambda^{(1)} = \begin{pmatrix} 0 & 0 & \sigma_3 \otimes A & 0 \\ \sigma_1 \otimes M_1 & 0 & 0 & 0 \\ 0 & -\sigma_3 \otimes M_2^{\mathrm{T}} & 0 & -\sigma_1 \otimes M_1^{\mathrm{T}} \\ \sigma_3 \otimes M_2 & 0 & 0 & 0 \end{pmatrix}, \tag{A.9}$$

where A is an antisymmetric 6×6 matrix and M_1 and M_2 are general 2×6 matrices. Under the gauge group $\mathrm{U}(6) \times \mathrm{Sp}(4)$, A transforms as $(15, 1)_{+2}$, while M_1 and M_2 form a $(\overline{6}, 4)_{-1}$, where the subscripts $+2$ and -1 denote the U(1) charges of the states. $\lambda^{(2)}$ and $\lambda^{(3)}$ have a similar structure, i.e. we get three copies of each of these states [11].

Introducing a Wilson line γ_W yields an additional constraint

$$\lambda^{(i)} = \gamma_W \lambda^{(i)} \gamma_W^{-1} \tag{A.10}$$

for those states that feel the Wilson line action. In the case of one Wilson line along the third two-torus, considered in the main text, this would only affect states in the 99-, $5_3 5_3$- and 95_3-sectors, since we assumed that all the D5-branes are located at the fixed point at the origin, where the Wilson line is not active.

For example, in the case of $\lambda^{(1)}$ from the 99-sector the matrices A, M_1 and M_2 from eq. (A.9) get reduced to

$$A = \begin{pmatrix} 0 & 0 \\ 0 & \tilde{A} \end{pmatrix}, \qquad M_1 = \begin{pmatrix} 0 & 0 \\ 0 & \tilde{M}_1 \end{pmatrix}, \qquad M_2 = \begin{pmatrix} 0 & 0 \\ 0 & \tilde{M}_2 \end{pmatrix}, \tag{A.11}$$

where $\tilde{A}$ is an antisymmetric 4×4 matrix and $\tilde{M}_1$ and $\tilde{M}_2$ are general 4-dimensional line-vectors. Under the gauge group $\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$, $\tilde{A}$ transforms as $(6, 1, 1, 1)(+2, 0)$ and $(\tilde{M}_1, \tilde{M}_2)$ form a $(\bar{4}, 1, 1, 2)(-1, 0)$, where the second set of brackets contains the U(1) charges of the fields.

A novel feature of this Wilson line action is that now the three matter Chan-Paton matrices in the 99- and $5_3 5_3$-sectors yield different matter states. Consider $\lambda^{(3)}$ from the 99-sector. In the absence of Wilson lines it will give the same matter states A, M_1 and M_2 as $\lambda^{(1)}$ (cf. eq. (A.9)). However, Wilson line action on $\lambda^{(3)}$ projects out fewer states than in the case of $\lambda^{(1)}$ and $\lambda^{(2)}$ and the surviving fields read

$$A = \begin{pmatrix} \tilde{A}_2 & 0 \\ 0 & \tilde{A} \end{pmatrix}, \qquad M_1 = \begin{pmatrix} \tilde{M}_3 & 0 \\ 0 & \tilde{M}_1 \end{pmatrix}, \qquad M_2 = \begin{pmatrix} \tilde{M}_4 & 0 \\ 0 & \tilde{M}_2 \end{pmatrix}, \tag{A.12}$$

where, in addition to the states $(6, 1, 1, 1)(+2, 0)$ and $(\bar{4}, 1, 1, 2)(-1, 0)$ discussed above, we have an antisymmetric 2×2 matrix $\tilde{A}_2$, corresponding to a $(1, 1, 1, 1)(0, +2)$, and two-dimensional line vectors $\tilde{M}_3$ and $\tilde{M}_4$ which form a $(1, 2, 2, 1)(0, -1)$.

B. Blowing-up modes from the $\mathbb{Z}_2$ twisted sectors in $\mathbb{Z}_3\times\mathbb{Z}_2\times\mathbb{Z}_2$ orbifolds

The physical moduli arising from each of the three $\mathbb{Z}_2$ twisted sectors have a similar form, we present the explicit result for the R_3 twist only. Let us denote the twist fields associated with the four ($i = 1, 2, 3, 4$) fixed points of the first two two-tori ($J = 1, 2$) under R_3 action as Σ_i^J. The blowing-up modes of the total 16 fixed points are represented by the fields $\omega_{i,j} = \Sigma_i^1\Sigma_i^2$. However, only the following six combinations of $\omega_{i,j}$ are invariant under the $\mathbb{Z}_3$ rotation:

$$\begin{aligned}
&\omega_{1,1}\,,\\
\omega_{\{\{2,3,4\}\},\{\{2,3,4\}\}^*} &\equiv \frac{1}{3}\left(\Sigma_2^1 + e^{2\pi i/3}\Sigma_3^1 + e^{4\pi i/3}\Sigma_4^1\right)\left(\Sigma_2^2 + e^{4\pi i/3}\Sigma_3^2 + e^{2\pi i/3}\Sigma_4^2\right),\\
\omega_{\{\{2,3,4\}\}^*,\{\{2,3,4\}\}} &\equiv \frac{1}{3}\left(\Sigma_2^1 + e^{4\pi i/3}\Sigma_3^1 + e^{2\pi i/3}\Sigma_4^1\right)\left(\Sigma_2^2 + e^{2\pi i/3}\Sigma_3^2 + e^{4\pi i/3}\Sigma_4^2\right),\\
\omega_{\{2,3,4\},1} &\equiv \frac{1}{\sqrt{3}}\left(\Sigma_2^1 + \Sigma_3^1 + \Sigma_4^1\right)\Sigma_1^2\,,\\
\omega_{1,\{2,3,4\}} &\equiv \frac{1}{\sqrt{3}}\Sigma_1^1\left(\Sigma_2^2 + \Sigma_3^2 + \Sigma_4^2\right),\\
\omega_{\{2,3,4\},\{2,3,4\}} &\equiv \frac{1}{3}\left(\Sigma_2^1 + \Sigma_3^1 + \Sigma_4^1\right)\left(\Sigma_2^2 + \Sigma_3^2 + \Sigma_4^2\right).
\end{aligned}$$

References

[1] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y.S. Stanev, *Chiral asymmetry in four-dimensional open-string vacua*, *Phys. Lett.* **B 385** (1996) 96 [hep-th/9606169].

[2] M. Berkooz and R.G. Leigh, *A $D = 4$ $N = 1$ orbifold of type-I strings*, *Nucl. Phys.* **B 483** (1997) 187 [hep-th/9605049].

[3] G. Zwart, *Four-dimensional $N = 1$ $\mathbb{Z}_n \times \mathbb{Z}_m$ orientifolds*, *Nucl. Phys.* **B 526** (1998) 378 [hep-th/9708040];
Z. Kakushadze, *Aspects of $N = 1$ type-I-heterotic duality in four dimensions*, *Nucl. Phys.* **B 512** (1998) 221 [hep-th/9704059];
Z. Kakushadze and G. Shiu, *A chiral $N = 1$ type-I vacuum in four dimensions and its heterotic dual*, *Phys. Rev.* **D 56** (1997) 3686 [hep-th/9705163];
Z. Kakushadze and G. Shiu, *4d chiral $N = 1$ type-I vacua with and without D5-branes*, *Nucl. Phys.* **B 520** (1998) 75 [hep-th/9706051];
L.E. Ibáñez, *A chiral $D = 4$, $N = 1$ string vacuum with a finite low energy effective field theory*, *J. High Energy Phys.* **07** (1998) 002 [hep-th/9802103];
D. O'Driscoll, *General abelian orientifold models and one loop amplitudes*, hep-th/9801114.

[4] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, *$D = 4$, $N = 1$, type-IIB orientifolds*, *Nucl. Phys.* **B 536** (1998) 29 [hep-th/9804026].

[5] L.E. Ibáñez, C. Munoz and S. Rigolin, *Aspects of type-I string phenomenology*, *Nucl. Phys.* **B 553** (1999) 43 [hep-ph/9812397].

[6] Z. Kakushadze and S.H. H. Tye, *Three generations in type-I compactifications*, *Phys. Rev.* **D 58** (1998) 126001 [hep-th/9806143];
G. Shiu and S.H. H. Tye, *Tev scale superstring and extra dimensions*, *Phys. Rev.* **D 58** (1998) 106007 [hep-th/9805157].

[7] A. Sen, *Stable non-BPS states in string theory*, *J. High Energy Phys.* **06** (1998) 007 [hep-th/9803194]; *Stable non-BPS bound states of BPS D-branes*, *J. High Energy Phys.* **08** (1998) 010 [hep-th/9805019]; SO(32) *spinors of type-I and other solitons on brane-antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141]; *Type-I D-particle and its interactions*, *J. High Energy Phys.* **10** (1998) 021 [hep-th/9809111].

[8] I. Antoniadis, E. Dudas and A. Sagnotti, *Brane supersymmetry breaking*, *Phys. Lett.* **B 464** (1999) 38 [hep-th/9908023].

[9] G. Aldazabal and A.M. Uranga, *Tachyon-free non-supersymmetric type-IIB orientifolds via brane-antibrane systems*, *J. High Energy Phys.* **10** (1999) 024 [hep-th/9908072].

[10] G. Aldazabal, L.E. Ibáñez and F. Quevedo, *Standard-like models with broken supersymmetry from type-I string vacua*, *J. High Energy Phys.* **01** (2000) 031 [hep-th/9909172].

[11] Z. Kakushadze, *A three-family* SU(6) *type-I compactification*, *Phys. Lett.* **B 434** (1998) 269 [hep-th/9804110];
A three-family $\mathrm{SU}(4)_c \otimes \mathrm{SU}(2)_w \otimes \mathrm{U}(1)$ *type-I vacuum*, *Phys. Rev.* **D 58** (1998) 101901 [hep-th/9806044];
Z. Kakushadze, *On four dimensional $N = 1$ type-I compactifications*, *Nucl. Phys.* **B 535** (1998) 311 [hep-th/9806008].

[12] M. Cvetič, L. Everett, P. Langacker and J. Wang, *Blowing-up the four-dimensional $\mathbb{Z}_3$ orientifold*, *J. High Energy Phys.* **04** (1999) 020 [hep-th/9903051].

[13] Z. Kakushadze, G. Shiu and S.H. H. Tye, *Type-IIB orientifolds with NS-NS antisymmetric tensor backgrounds*, *Phys. Rev.* **D 58** (1998) 086001 [hep-th/9803141].

[14] P. Hořava, *Strings on world sheet orbifolds*, *Nucl. Phys.* **B 327** (1989) 461.

[15] D. Friedan, E. Martinec and S. Shenker, *Conformal invariance, supersymmetry and string theory*, *Nucl. Phys.* **B 271** (1986) 93.

[16] L.E. Ibáñez, R. Rabadan and A.M. Uranga, *Anomalous* U(1)*'s in type-I and type-IIB $D = 4$, $N = 1$ string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139].

[17] M.R. Douglas and G. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.

[18] I. Antoniadis, C. Bachas and E. Dudas, *Gauge couplings in four-dimensional type-I string orbifolds*, *Nucl. Phys.* **B 560** (1999) 93 [hep-th/9906039].

[19] Z. Lalak, S. Lavignac and H.P. Nilles, *String dualities in the presence of anomalous* U(1) *symmetries*, *Nucl. Phys.* **B 559** (1999) 48 [hep-th/9903160].

[20] M. Klein, *Anomaly cancellation in $d = 4$, $N = 1$ orientifolds and linear/chiral multiplet duality*, *Nucl. Phys.* **B 569** (2000) 362 [hep-th/9910143].

[21] G. Aldazabal, D. Badagnani, L.E. Ibáñez and A.M. Uranga, *Tadpole versus anomaly cancellation in $D = 4, 6$ compact IIB orientifolds*, *J. High Energy Phys.* **06** (1999) 031 [hep-th/9904071].